Logical Forms

Lecture 1
Section 2.4

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Outline

Statements

- A statement is a sentence that is either true or false, but not both.
- These are statements:
  - Today is Saturday.
  - Discrete Math meets today.
- These are *not* statements:
  - Hello.
  - Are you there?
  - Go away!

Outline

Logical Operators

- Binary operators
  - Conjunction – “and”
  - Disjunction – “or”
- Unary operator
  - Negation – “not”
- Other operators
  - XOR – “exclusive or”
  - NAND – “not both”
  - NOR – “neither”

Logical Symbols

- Statements are represented by letters: p, q, r, etc.
- $\wedge$ means “and”.
- $\vee$ means “or”.
- $\sim$ means “not”.

Examples

- Basic statements
  - p = “HSC is in North Carolina.”
  - q = “UVA is in Virginia.”
- Compound statements
  - $p \wedge q =$ “HSC is in North Carolina and UVA is in Virginia.”
  - $p \vee q =$ “HSC is in North Carolina or UVA is in Virginia.”
  - $\sim p =$ “HSC is not in North Carolina.”
  - $(\sim p) \wedge (\sim q)$ = ???
  - $\sim (p \wedge q)$ = ???

False Negations

- Statement: “Everyone loves Raymond.”
- Incorrect negation: “Everyone does not love Raymond.”
- Correct negation: “Someone does not love Raymond.”

False Negations

- Statement: “Someone loves Raymond.”
- Incorrect negation: “Someone does not love Raymond.”
- Correct negation: “No one loves Raymond.”

Outline

Truth Table of an Expression

- Make a column for every variable.
- List every possible combination of truth values of the variables.
- Make one more column for the expression.
- Write the truth value of the expression for each combination of truth values of the variables.

Truth Table for "and"

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- $p \wedge q$ is true if p is true *and* q is true.
- $p \wedge q$ is false if p is false *or* q is false.

Truth Table for “or”

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- $p \vee q$ is true if p or true *and* q is true.
- $p \vee q$ is false if p and false *or* q is false.

Truth Table for “not”

p	$\sim p$
T	F
F	T

- $\sim p$ is true if p is false.
- $\sim p$ is false if p is true.

Example: Truth Table

p	q	r	$(\sim p) \vee (q \wedge r)$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

- Truth table for the statement $(\sim p) \vee (q \wedge r)$.

Outline

Assignment

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- Read Section 2.1, pages 23 - 29.
- Exercises 6 - 8, 10, 12 - 15, page 37.